

## Reductieregels

$$\mathbf{X + X.Y = X}$$

$$\mathbf{Bewijs : X.1 + X.Y = X(\cancel{1+Y})}$$

|  
1

$$\mathbf{X + \bar{X}.Y = X + Y}$$

$$\mathbf{Bewijs : (\cancel{X + \bar{X}}).(X + Y) = 1.(X + Y)}$$

|  
1

$$\mathbf{X.(X + Y) = X}$$

$$\mathbf{Bewijs : X.1 + X.Y = X(\cancel{1+Y})}$$

|  
1

$$\mathbf{X.(\bar{X} + Y) = X.Y}$$

$$\mathbf{Bewijs : \cancel{X.\bar{X}} + X.Y = 0 + A.B}$$

|  
0

# VEREENVOUDIGINGSTECHNIEKEN

## 3.4.1 Vereenvoudigingsregels

Voorbeeld 1

$$\begin{aligned} X &= BCD + B\bar{D}\bar{E} + \bar{B}\bar{E} + \bar{B}\bar{D}CD + B\bar{E} + B\bar{D}E + \bar{C}\bar{D}\bar{E} + \bar{C}E \\ &\quad (\bar{B}\bar{D}CD=0) \text{ omdat } D\bar{D}=0 \end{aligned}$$

$$\begin{aligned} X &= BCD + B\bar{D}\bar{E} + \bar{B}\bar{E} + B\bar{E} + B\bar{D}E + \bar{C}\bar{D}\bar{E} + \bar{C}E \\ &\quad (\bar{B}\bar{E} + B\bar{E} = \bar{E}) \rightarrow (\bar{B} + B)\bar{E} = \bar{E} \end{aligned}$$

$$\begin{aligned} X &= BCD + B\bar{D}\bar{E} + \bar{E} + B\bar{D}E + \bar{C}\bar{D}\bar{E} + \bar{C}E \\ &\quad (\bar{E} + B\bar{D}\bar{E} + \bar{C}\bar{D}\bar{E} = \bar{E}) \rightarrow (1 + B\bar{D} + \bar{C}\bar{D})\bar{E} \end{aligned}$$

$$\begin{aligned} X &= BCD + \bar{E} + B\bar{D}E + \bar{C}E \\ &\quad (\bar{E} + \bar{C}E = \bar{E} + \bar{C}) \rightarrow (\bar{E} + \bar{C})(\bar{E} + E) \end{aligned}$$

$$\begin{aligned} X &= BCD + \bar{E} + B\bar{D}E + \bar{C} \\ &\quad (\bar{E} + B\bar{D}E = \bar{E} + B\bar{D}) \rightarrow (\bar{E} + B\bar{D})(\bar{E} + E) \end{aligned}$$

$$\begin{aligned} X &= BCD + \bar{E} + B\bar{D} + \bar{C} \\ &\quad (BCD + \bar{C} = BD + \bar{C}) \rightarrow (BD + \bar{C})(C + \bar{C}) \end{aligned}$$

$$\begin{aligned} X &= BD + \bar{E} + B\bar{D} + \bar{C} \\ &\quad (BD + B\bar{D} = B) \rightarrow B(D + \bar{D}) \end{aligned}$$

$$X = B + \bar{C} + \bar{E}$$

## Voorbeeld 2

$$\mathbf{X} = \overline{\overline{\overline{A + B + C}} \overline{A + B + C + \overline{AC}}} \\ \text{(De Morgan)}$$

$$\mathbf{X} = \overline{\overline{\overline{A + B + C}} \overline{A + B + C} \bullet \overline{AC}} \\ \text{(De Morgan)}$$

$$\mathbf{X} = \overline{\overline{\overline{A + B + C} + A + \overline{B} + C} \overline{AC}} \\ \text{(De Morgan)}$$

$$\mathbf{X} = \overline{\overline{A + \overline{B} \bullet C} + A + \overline{B} + C} \overline{AC} \\ (A + A = A)$$

$$\mathbf{X} = \overline{A + \overline{B} \bullet C + \overline{B} + C} \overline{AC} \\ (\overline{B}C + \overline{B} = \overline{B})$$

$$\mathbf{X} = \overline{A + \overline{B} + C} \overline{AC} \\ (A \bullet \overline{AC} = 0)$$

$$\mathbf{X} = \overline{A} \overline{B} C + \overline{A} C C \\ (\overline{A} C C = \overline{A} C)$$

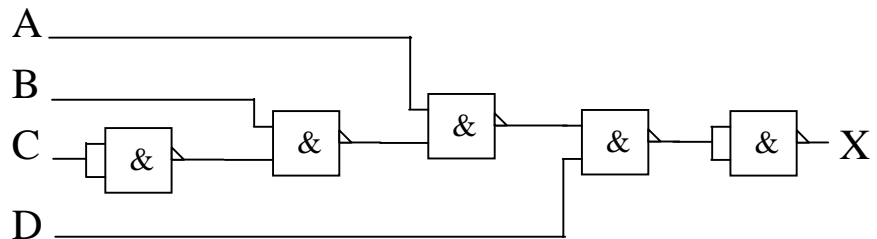
$$\mathbf{X} = \overline{A} \overline{B} C + \overline{A} C$$

$$\mathbf{X} = \overline{A} C$$

## OEFENINGEN IMPLEMENTATIE

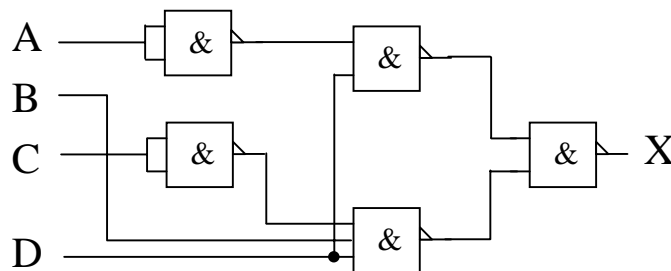
Uitsluitend NAND – poorten

$$\begin{aligned}
 X &= (\overline{\overline{A}} + \overline{BC}) \cdot D \\
 &= \overline{\overline{\overline{A}} + \overline{BC}} \cdot D \\
 &= \overline{\overline{A} \cdot \overline{BC}} \cdot D \\
 &= \overline{A \cdot B \cdot C} \cdot D
 \end{aligned}$$

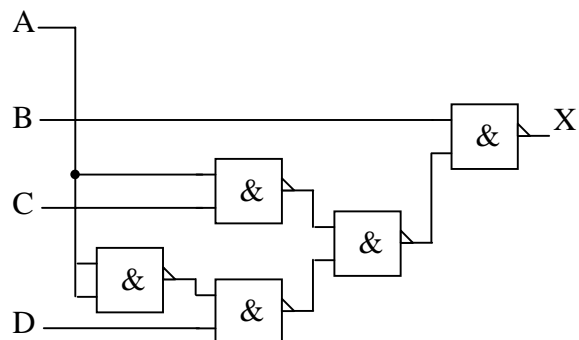


(eerst de functie vereenvoudigen)

$$\begin{aligned}
 X &= (\overline{A} + \overline{BC}) \cdot D \\
 &= \overline{AD} + \overline{BCD} \\
 &= \overline{\overline{\overline{AD}} + \overline{\overline{BCD}}} \\
 &= \overline{\overline{AD} \cdot \overline{BCD}}
 \end{aligned}$$

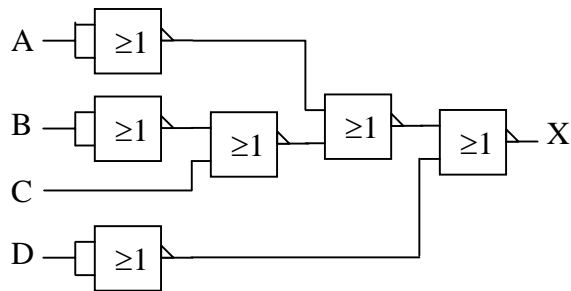


$$\begin{aligned}
 X &= \overline{\overline{ABD} + \overline{BC(A + D)}} \\
 &= \overline{\overline{ABD} + \overline{ABC} + \overline{BCD}} \\
 &= \overline{B(\overline{AD} + \overline{AC} + \overline{CD})} \\
 &= \overline{B(\overline{AD} + \overline{AC} + \overline{CD(A + \overline{A})})} \\
 &= \overline{B(\overline{AD} + \overline{ACD} + \overline{AC} + \overline{ACD})} \\
 &= \overline{B(\overline{AD} + \overline{AC})} \\
 &= \overline{B \cdot \overline{\overline{AD}} \cdot \overline{\overline{AC}}}
 \end{aligned}$$



## Uitsluitend NOR – poorten

$$\begin{aligned}
 X &= (\bar{A} + B\bar{C})D \\
 &= \overline{\overline{\bar{A} + B\bar{C}}}D \\
 &= \overline{\overline{\bar{A} + \bar{B} + C}}D \\
 &= \overline{\bar{A} + \bar{B} + C + \bar{D}}
 \end{aligned}$$



(eerst de functie vereenvoudigen)

$$\begin{aligned}
 X &= (\bar{A} + B\bar{C})D \\
 &= \overline{\bar{A}D} + \overline{BCD} \\
 &= \overline{\bar{A} + \bar{D}} + \overline{\bar{B} + C + \bar{D}}
 \end{aligned}$$

Deze schakeling is iets complexer

$$\begin{aligned}
 X &= A\bar{C}(B + \bar{D}) + \bar{B}\bar{C}\bar{D} \\
 &= \bar{C}(A(B + \bar{D}) + \bar{B}\bar{D}) \\
 &= \bar{C}(AB + A\bar{D} + \bar{B}\bar{D}) \\
 &= \bar{C}(AB + A\bar{D}(B + \bar{B}) + \bar{B}\bar{D}) \\
 &= \bar{C}(AB + AB\bar{D} + A\bar{B}\bar{D} + \bar{B}\bar{D}) \\
 &= \bar{C}(AB(1 + \bar{D}) + \bar{B}\bar{D}(1 + A)) \\
 &= \bar{C}(AB + \bar{B}\bar{D}) \\
 &= \bar{C}(\overline{\bar{A} + \bar{B}} + \overline{\bar{B} + D}) \\
 &= \overline{\overline{C} + \bar{A} + \bar{B} + \bar{B} + D}
 \end{aligned}$$

## OEFENINGEN OP DE VEREENVOUDIGINGSREGELS

1.

$$\begin{aligned}x &= abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c} \\ &= ab(c + \bar{c}) + a\bar{b}(c + \bar{c}) \\ &= ab + a\bar{b} \\ &= a(b + \bar{b}) \\ &= a\end{aligned}$$

2.

$$\begin{aligned}x &= \bar{a}\bar{b} + \bar{a}b + a\bar{b} \\ &= \bar{a}(\bar{b} + b) + a\bar{b} \\ &= \bar{a} + a\bar{b} \\ &= \bar{a} + \bar{b}\end{aligned}$$


3.

$$\begin{aligned}x &= abc + ab\bar{c} + \bar{a}bc \\ &= ab(c + \bar{c}) + \bar{a}bc \\ &= ab + \bar{a}bc \\ &= b(a + \bar{a}c) \\ &= b(a + c)\end{aligned}$$

4.

$$\begin{aligned}x &= a\bar{b} + c + \bar{a}\bar{c}d + b\bar{c}d \\ &= a\bar{b} + [c + \bar{c}(\bar{a}d + bd)] \\ &= a\bar{b} + c + \bar{a}d + bd \\ &= a\bar{b} + c + d(\bar{a} + b) \\ &= a\bar{b} + c + d(\overline{a\bar{b}}) \\ &= a\bar{b} + c + d\end{aligned}$$

$\rightarrow X + \bar{X}Y = X + Y$



# OEFENINGEN MET KARNAUGHKAARTEN

$$F = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{C}\overline{D} + \overline{A}B\overline{D} + B\overline{C}\overline{D} + ABC$$

F		CD			
		00	01	11	10
AB	00	1	1	0	0
	01	1	1	1	1
	11	0	0	1	1
	10	0	0	0	0

$$F = \overline{A}\overline{C} + BC$$

$$F = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D}$$

F		CD			
		00	01	11	10
AB	00	1	0	0	1
	01	0	0	0	0
	11	0	0	0	0
	10	1	0	0	1

$$F = \overline{B}\overline{D}$$